Incentivizing Effort in Interdependent Security Games Using Resource Pooling

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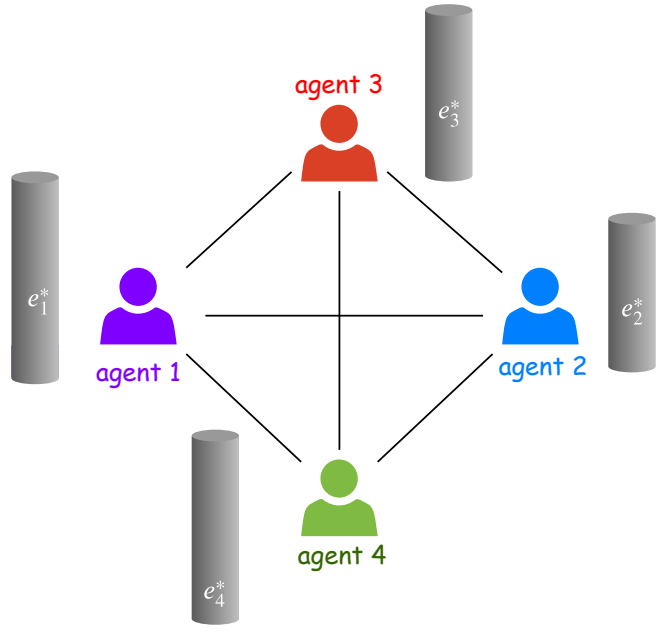
June 30, 2019

InterDependent Security (IDS) Games



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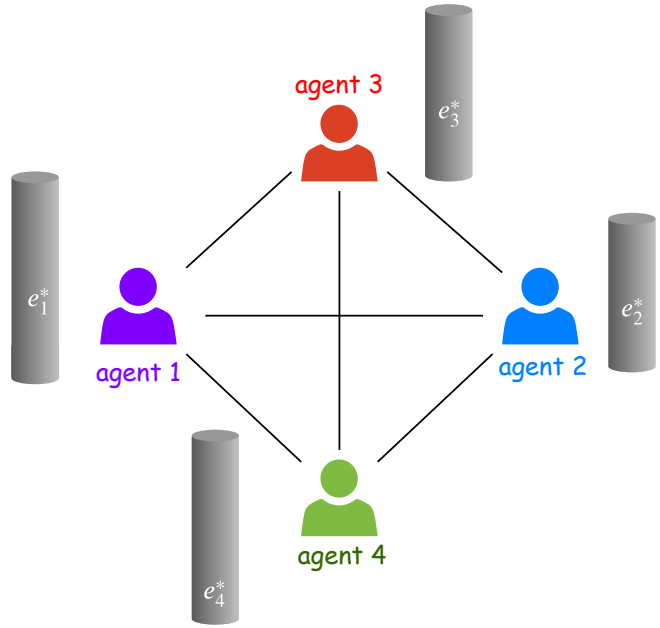

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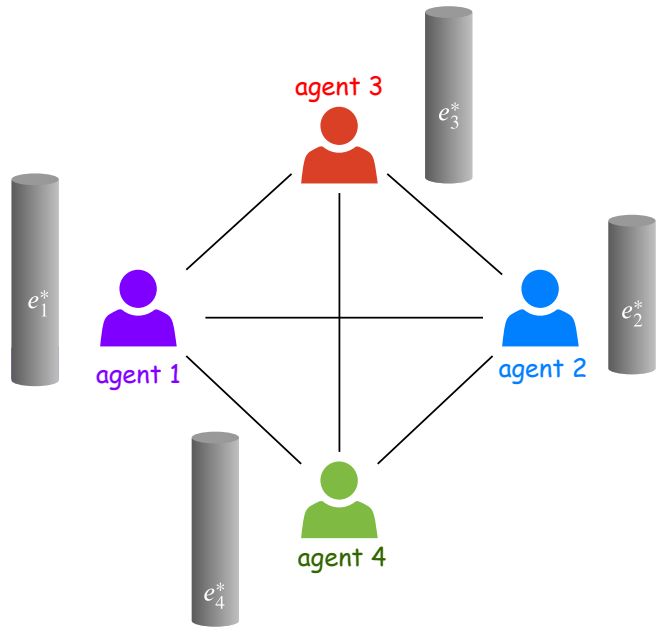
- Actions: Security investment/effort e_i



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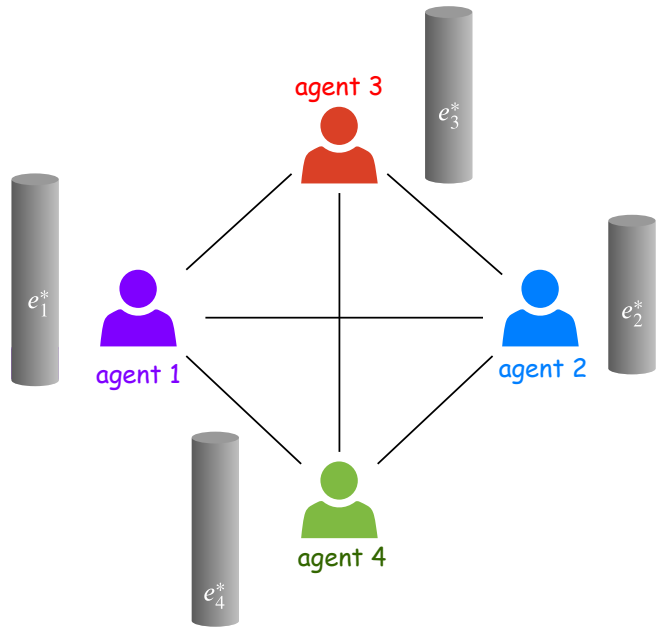

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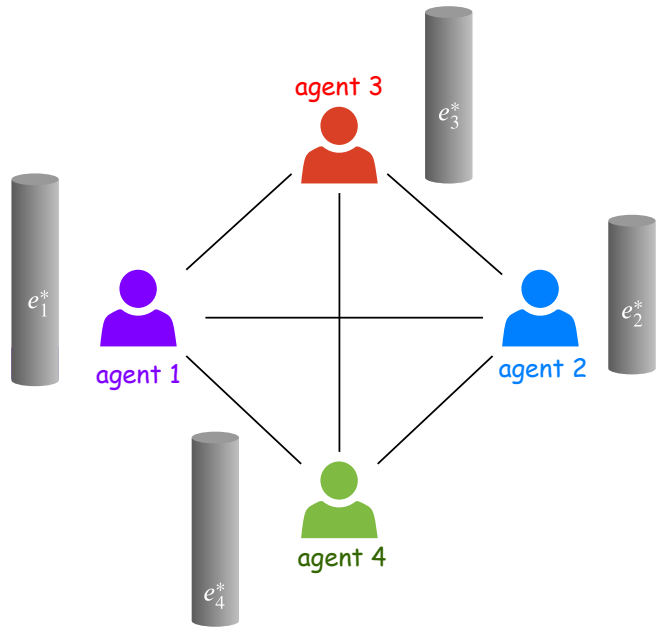

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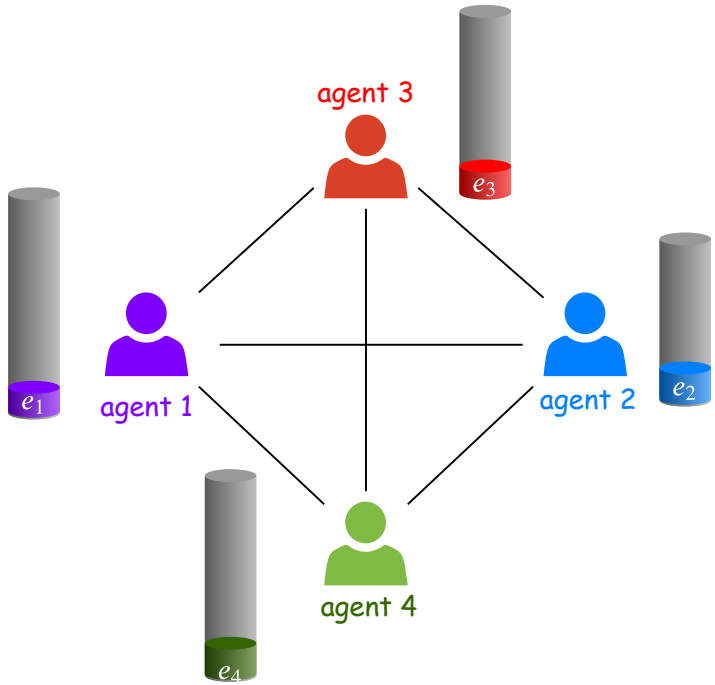

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- JPMorgan Chase: one of the servers did not have 2 factors authentication

Addressing Under-investment Issue

Traditional Solutions in the literature of IDS games:

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- Incentive/Taxation Mechanism [1]

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  - Costly due to the social/privacy reasons

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Examples of Resource pooling,

- Funding an open source project
- Security product discount

Overview

Motivation

IDS Game Without Resource Pooling

IDS Game with Resource Pooling

Voluntary Participation

Community Based Resource Pooling

Conclusion

Model: IDS Game without Resource Pooling

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- Agent i's utility

$$u_i(e_i, e_{-i}) = -l_i + a_i e_i - b_i e_i^2 + e_i \sum_j x_{ij} e_j$$

No Resource Pooling: Nash Equilibrium

Theorem

Assume $2b_i \geq \sum_j x_{ij}, \forall i$. *Then, IDS game without resource pooling has the unique NE* ($\hat{\boldsymbol{e}} = [\hat{e}_1, \hat{e}_2, \cdots, \hat{e}_n]$).

No Resource Pooling: Socially Optimal Effort Level

- Socially Optimal Effort level

$$\boldsymbol{e}^* = \arg \max_{\boldsymbol{e}=[e_1,e_2,\cdots e_n]} \sum_i u_i(\boldsymbol{e})$$

Theorem

Assume $2b_i \geq \sum_j x_{ij} + x_{ji}, \forall i$. *Then, socially optimal effort level* $\boldsymbol{e}^*$ *is unique. Moreover,* $e_i^* > \hat{e}_i, \ \forall i$.

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$$v_i(\boldsymbol{e}_i, \boldsymbol{e}_{-i}) = -l_i + a_i E_i + E_i \cdot \sum_{j=1}^{n} x_{ij} E_j - \sum_{k=1}^{n} b_k \cdot e_{ik}^2.$$

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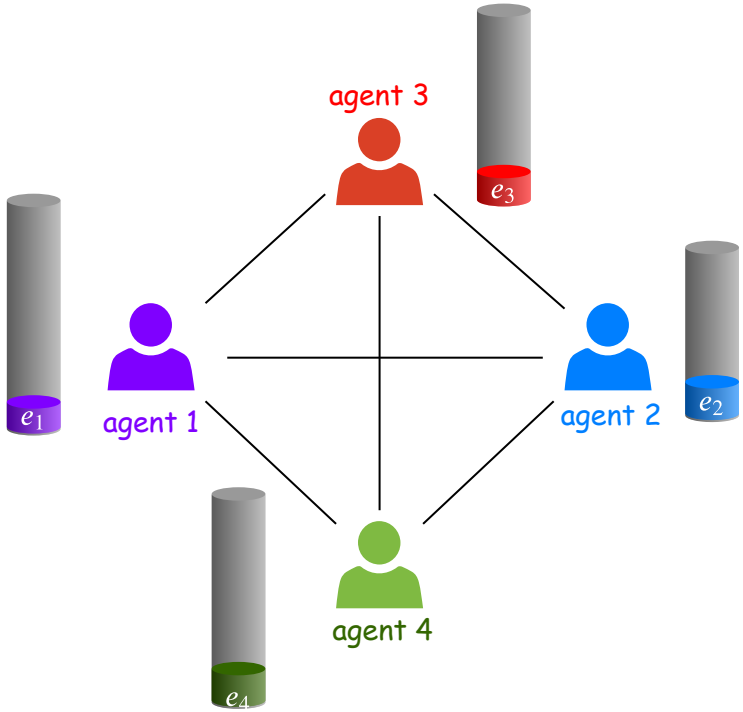
Let $\hat{E}_i$ *be the total effort exerted on behalf of agent* i *at the NE.*
Then, $\begin{bmatrix} \hat{E}_1 \\ \vdots \\ \hat{E}_n \end{bmatrix} = \boldsymbol{e}^*$ *is the total effort at the NE of the game with resource pooling.*

Resource Pooling: Nash Equilibrium

IDS game **without** resource pooling



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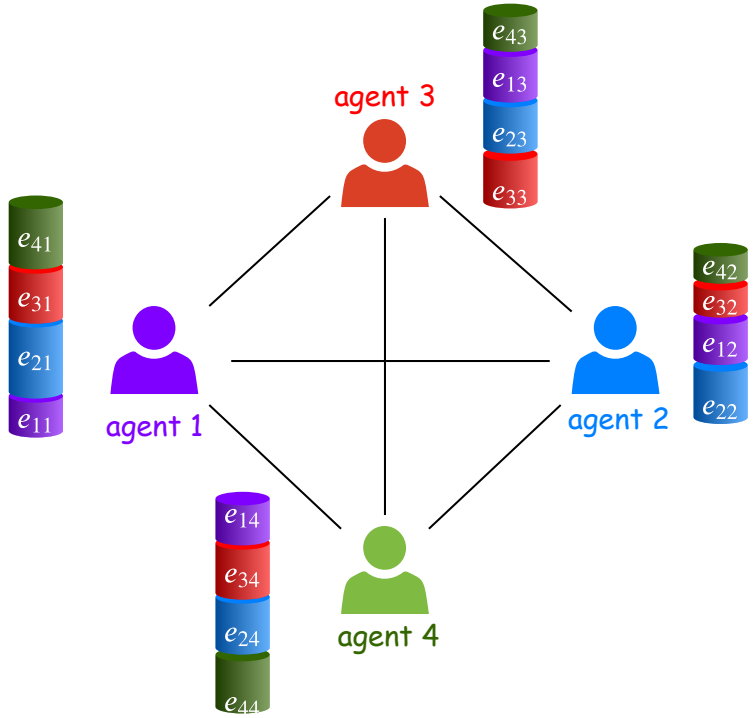

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- $v_i(\hat{E}) \geq u_i(\hat{e}_i, \hat{e}_{-i})$
- $\sum_{i=1}^{n} v_i(\hat{E}) \geq \sum_{i=1}^{n} u_i(\boldsymbol{e}^*)$

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- Let $\overline{E} = [\overline{e}_{ij}]_{n \times n}$ be the NE of game G^k and $v_k(\overline{E})$ be the utility of agent i at the NE.
- We say that resource pooling has the voluntary participation property with respect to agent k, if

$$v_k(\overline{E}) \leq v_k(\hat{E}), \tag{1}$$

where $\hat{E}$ is the effort profile at the NE of game with resource pooling.

Voluntary Participation

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Resource Pooling always satisfies the Voluntary Participation with respect to all agents.

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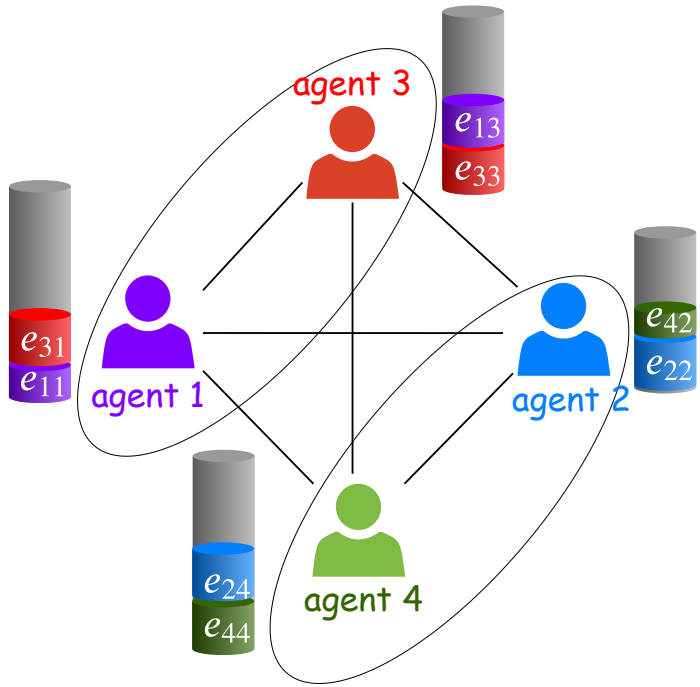
Agents form communities $C_1, C_2, \cdots, C_m$ and are allowed to pool resources within their own communities. $C_i \cap C_j = \emptyset$.



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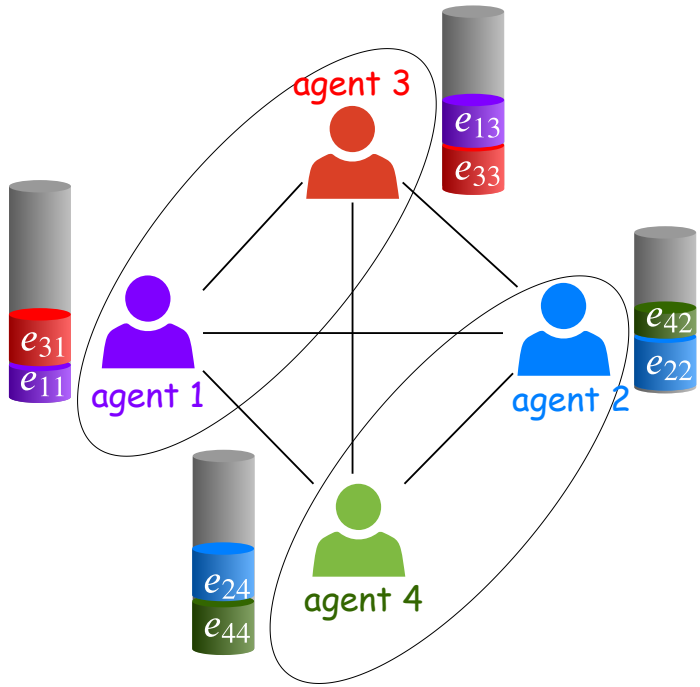

Theorem (Informal)

Community Based Resource Pooling improves agents' utilities and their efforts.

Resource Pooling within Communities: example

$n = 10$ agents in the network

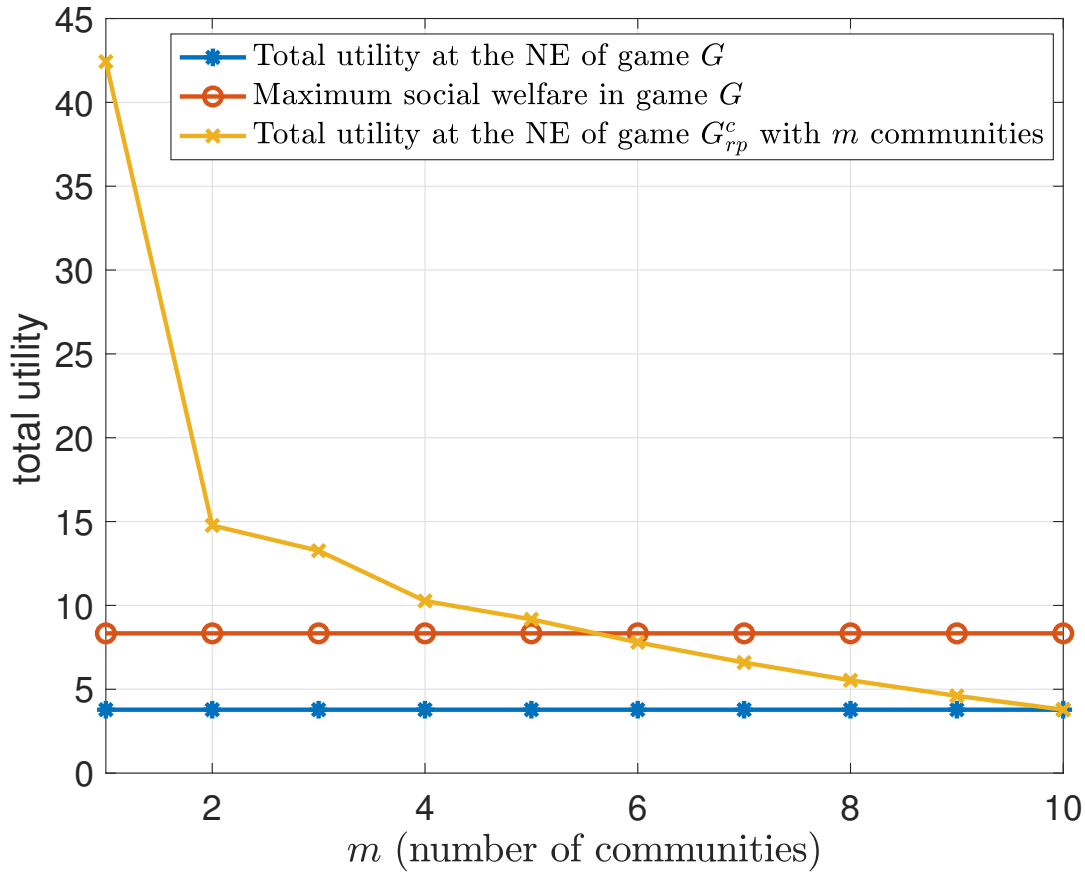

Figure: Total utility as a function of number of communities

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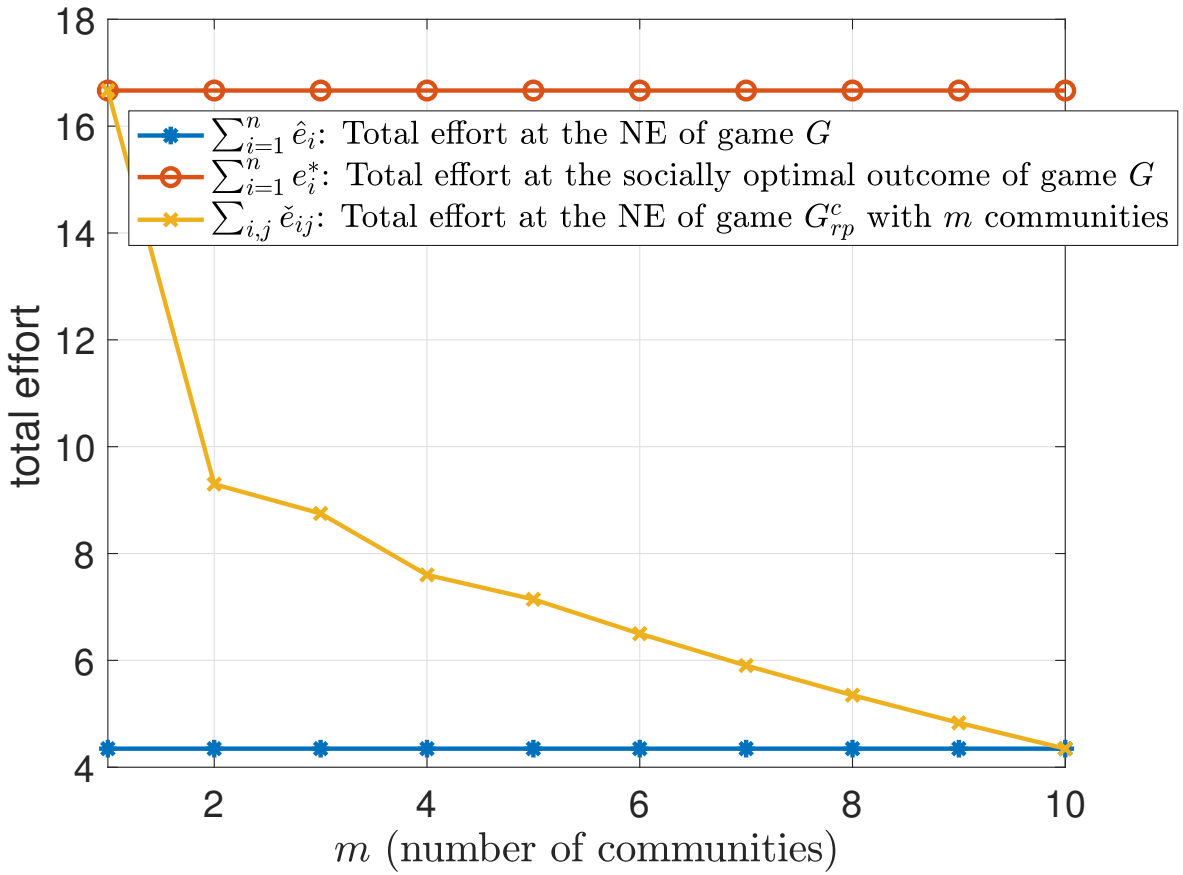

Figure: Total effort as a function of number of communities.

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- resource pooling increases the total effort exerted on behalf of each agent as compared to no resource pooling
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- Social welfare at the NE of the game with resource pooling is higher than the optimal social welfare under the game without resource pooling
- agents voluntarily participate in resource pooling

Future Work

- Study resource pooling with non-quadratic
- Resource pooling may not help under other models
  - With the limited effort budget, resource pooling may not help[3]

[3] Khalili, Mohammad Mahdi, Xueru Zhang, and Mingyan Liu. "Public Good Provision Games on Networks with Resource Pooling." Network Games, Control, and Optimization. Birkhuser, Cham, 2019. 271-287.

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