Contract Design for Purchasing Private Data Using a Biased Differentially Private Algorithm

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Motivation

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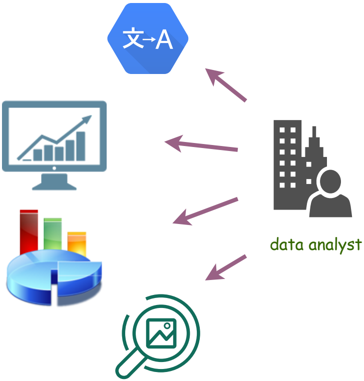

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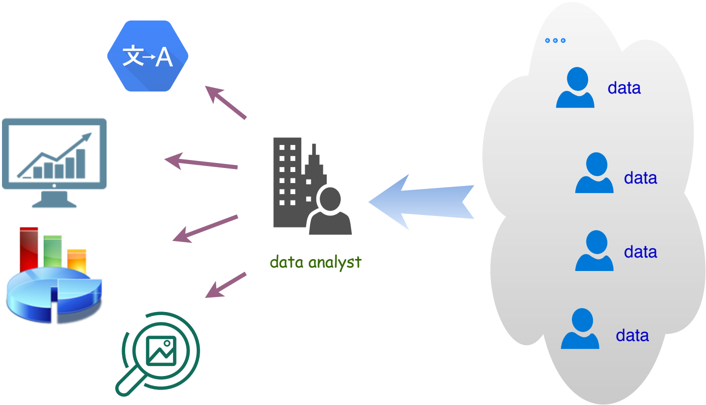

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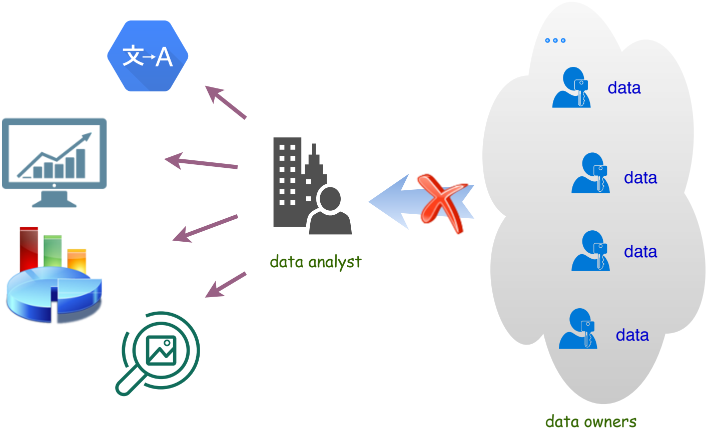

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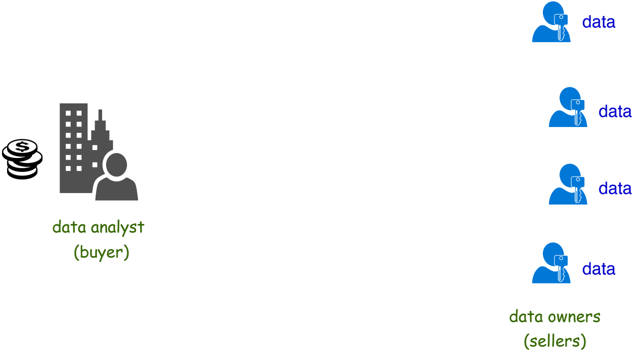

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data
data
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data
data analyst
(buyer)
data broker
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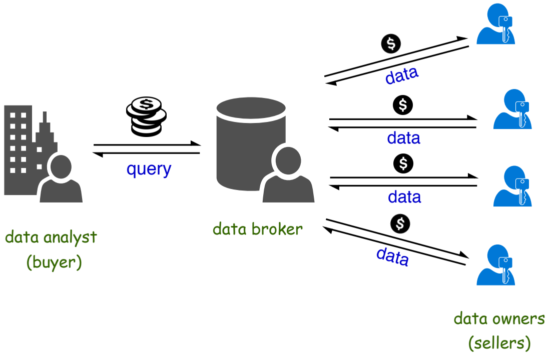
query
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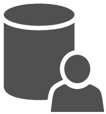

- Private algorithm to generate query $\mathcal{Q}$
- Optimal contract to minimize buyer's payment:
  - The data owners are compensated properly to share their data

An example

Datacoup: A new startup founded in 2012 and plays a role as a data broker

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- Datacoup offers a fixed monthly payment for having access to users' social media activities, credit card transactions, etc.
- Provides various computations for data analysts
- Datacoup removes identifiable markers

Related work

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- Proposed contracts using our private algorithm improves the payment-accuracy tradeoff.
- We extend the algorithm to multi-dimensional data and non-linear queries.

Model

- Database $D = (d_1, d_2, \cdots, d_n)$ with $d_i \in [0, 1]$. d_i belongs to individual/seller i.
- Query $\mathcal{Q} : [0, 1]^n \to \mathbb{R}$: $\mathcal{Q}(D) = \sum_{i=1}^{n} d_i$.

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- Obtain almost the same conclusion regardless of participation

Student ID	Last Name	Initial	Age	Program
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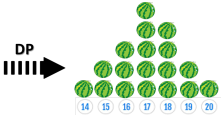

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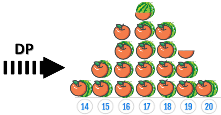

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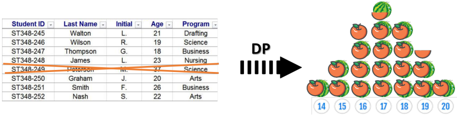

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- A randomized algorithm $\mathcal{A}(\cdot)$ is ϵ_i-differentially private w.r.t. **individual** i if for any two datasets $D^{(i)}$, $\hat{D}^{(i)}$ differing in i's data and for any sets of possible outputs $S \subseteq \text{range}(\mathcal{A})$:

$$\frac{\Pr(\mathcal{A}(D^{(i)}) \in S)}{\Pr(\mathcal{A}(\hat{D}^{(i)}) \in S)} \leq \exp(\epsilon_i), \ \ \epsilon_i \in [0, +\infty)$$

Model

K-accurate algorithm

$\mathcal{A}(D)$ is K-accurate for query $\mathcal{Q}(D)$ if its mean squared error (MSE) is at most K for all $D \in [0,1]^n$:

$$\mathbb{E}[||\mathcal{A}(D) - \mathcal{Q}(D)||^2] \leq K, \ \forall D \in [0,1]^n$$

Relationship between K and $\epsilon = \sum_{i=1}^{n} \epsilon_i$

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Theorem

A lower bound on total privacy loss $\epsilon = \sum_{i=1}^{n} \epsilon_i$: Consider an algorithm $\mathcal{A}(D)$ that is K-accurate for $\mathcal{Q}(D)$:

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- *if $K < (\frac{n}{2})^2$, then $\epsilon = \sum_{i=1}^{n} \epsilon_i \geq \ln \frac{(n-\sqrt{K})^2}{K} = \epsilon^{lb}$.*

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- *if $K < (\frac{m}{2})^2$, let $\mathcal{S} = \{i | \epsilon_i > 0, i = 1, 2, \cdots n\}$, then $|\mathcal{S}| \geq n - m + 1$.*

Two differentially private algorithms

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An unbiased algorithm (Laplace mechanism):

$$\mathcal{A}_u(D) = \sum_{i=1}^{n} d_i + N(b)$$

$$\boxed{f_N(x; b) = \frac{1}{2b} \exp\{-\frac{|x|}{b}\}}$$

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A biased algorithm:

$$\mathcal{A}_{new}(D) = \sum_{i=1}^{n} a_i d_i + \sum_{i=1}^{n} \frac{1 - a_i}{2} + N(b), \quad a_i \in [0, 1], \forall i$$

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	privacy ϵ_i	accuracy K	bias $\lvert\mathbb{E}[\mathcal{A}(D) - \mathcal{Q}(D)]\rvert$
$\mathcal{A}_u(D)$	$1/b$	$2b^2$	0
$\mathcal{A}_{new}(D)$	a_i/b	$(\sum_{i=1}^{n} \frac{1-a_i}{2})^2 + 2b^2$	$\lvert\sum_{i=1}^{n}(a_i-1)d_i + \frac{1-a_i}{2}\rvert \leq \sum_{i=1}^{n} \frac{1-a_i}{2}$

Under full information: *a single seller* $D = d$

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- Full information: v is known to broker and buyer
- If the seller receives a payment (p) more than his privacy cost, he will share his own data
  - $p \geq c(v, \epsilon)$: Individual Rationality (IR)

Under full information: *a single seller* $D = d$

Contract design problem: Finding Contract (p, ϵ, K)

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Under full information: *a single seller $D = d$*

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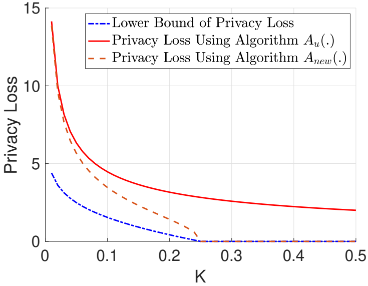
$\mathcal{A}_{new}(D) = a \cdot d + \frac{1-a}{2} + N(b)$

$$\min_{\{a \in [0,1], b>0, p\}} \quad p$$
$$s.t.(IR) \; p \geq c(v, \epsilon)$$
$$(AC) \; (\frac{1-a}{2})^2 + 2b^2 = K$$
$$\epsilon = \frac{a}{b}$$

Under full information: *a single seller $D = d$*



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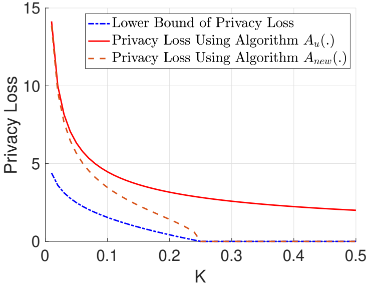

- $\mathcal{A}_{new}(D)$ outperforms $\mathcal{A}_u(D)$ in terms of privacy(payment)-accuracy tradeoff.

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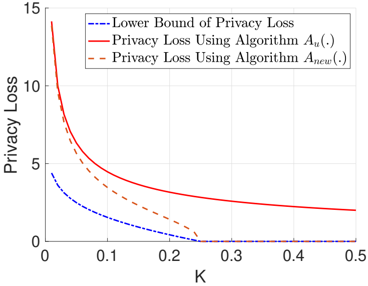


- $\mathcal{A}_{new}(D)$ outperforms $\mathcal{A}_u(D)$ in terms of privacy(payment)-accuracy tradeoff.
- when accuracy requirement is low ($K > 1/4$): best strategy is to report pure noise.

Under full information: n sellers $D = (d_1, d_2, \cdots, d_n)$

$\mathcal{A}_{new}(D) = \sum_{i=1}^{n} a_i \cdot d_i + \frac{1-a_i}{2} + N(b)$

$$\min_{\{a_i \in [0,1], b>0, p_i, i=1,\cdots,n\}} \sum_{i=1}^{n} p_i$$

$$s.t. (IR)\ p_i \geq c(v_i, \frac{a_i}{b}), i = 1, 2, \cdots n$$

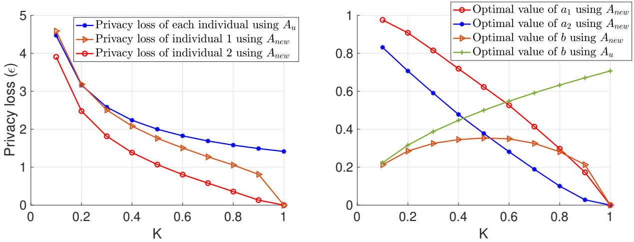
$$(AC)\ (\sum_{i=1}^{n} \frac{1-a_i}{2})^2 + 2b^2 = K$$

Theorem

The optimal solution under linear cost: *If $c(v_i, \epsilon_i) = v_i \cdot \epsilon_i\ \forall i$, then there is a closed form solution to contract design problem.*

A numerical example:

- Two sellers: $v_1 = 5$, $v_2 = 10$, $c(v_i, \epsilon_i) = v_i(e^{\epsilon_i} - 1)$



  – personalized privacy: $\mathcal{A}_{new}(D)$ assigns different ϵ_i to different individuals.
  – $\mathcal{A}_{new}(D)$ adds less noise than $\mathcal{A}_u(D)$: better privacy-accuracy tradeoff.

Motivation

Model

Private Algorithms

Contract under Full Information

Contract under Information Asymmetry

Conclusion

Under information asymmetry: *unknown privacy valuation*

- Two sellers $D = (d_1, d_2)$
- Privacy valuation is binary distributed:

$$v_i = \begin{cases} v_H, & w.p.\ \pi \\ v_L, & w.p.\ 1-\pi \end{cases}, \quad v_H > v_L$$

Design a **menu** of contracts $\{(p_H, \epsilon_H, K), (p_L, \epsilon_L, K)\}$:

- Incentive compatibility (IC):

$$p_H - c(v_H, \epsilon_H) \geq p_L - c(v_H, \epsilon_L) \ ,$$
$$p_L - c(v_L, \epsilon_L) \geq p_H - c(v_L, \epsilon_H) \ .$$

- Two options:
  - Offering **both** sellers the menu of contracts.
  - Offering only **one** seller the menu of contracts.

Option 1: offering both sellers the menu of contracts

Due to the uncertainty of v_1, v_2:

$$
\begin{aligned}
A_{new}(D) &= a_1 d_1 + a_2 d_2 + \frac{1-a_1}{2} + \frac{1-a_2}{2} + N(b), a_i \in \{a_H, a_L\} \\
&= \begin{cases}
a_H d_1 + a_H d_2 + \frac{1-a_H}{2} + \frac{1-a_H}{2} + N(b) \text{ w.p. } \pi^2 \\
a_H d_1 + a_L d_2 + \frac{1-a_H}{2} + \frac{1-a_L}{2} + N(b) \text{ w.p. } \pi(1-\pi) \\
a_L d_1 + a_H d_2 + \frac{1-a_L}{2} + \frac{1-a_H}{2} + N(b) \text{ w.p. } \pi(1-\pi) \\
a_L d_1 + a_L d_2 + \frac{1-a_L}{2} + \frac{1-a_L}{2} + N(b) \text{ w.p. } (1-\pi)^2
\end{cases}
\end{aligned}
$$

- $\epsilon_H = \frac{a_H}{b}$, $\epsilon_L = \frac{a_L}{b}$
- Expected accuracy

$$
\begin{aligned}
e(a_L, a_H, b) &= \pi^2 \cdot (2b^2 + (1-a_H)^2) + (1-\pi)^2 \cdot (2b^2 + (1-a_L)^2) \\
&+ 2\pi \cdot (1-\pi) \cdot (2b^2 + ((1-a_H)/2 + (1-a_L)/2)^2)
\end{aligned}
$$

Option 1: offering both sellers the menu of contracts

Design the menu of contracts:

$$
\begin{aligned}
\min_{\{p_i, a_i, b\}, i \in \{H,L\}} \quad & \mathbb{E}(p) = \pi^2 \cdot (2p_H) + (1-\pi)^2 \cdot (2p_L) + 2\pi(1-\pi) \cdot (p_H + p_L) \\
\text{s.t.} \qquad (IR) \quad & p_i \geq c(v_i, a_i/b), \;\; i \in \{H, L\} \\
(IC) \quad & p_i - c(v_i, a_i/b) \geq p_j - c(v_i, a_j/b), \;\; i, j \in \{H, L\} \\
(AC) \quad & e(a_L, a_H, b) \leq K, \;\; i \in \{H, L\} \\
& 0 \leq a_i \leq 1, p_i \geq 0, b > 0, \;\; i \in \{H, L\}
\end{aligned}
$$

Option 2: offering only one seller the menu of contracts

Due to the uncertainty of v_1, v_2:

$$A_{new}(D) = \begin{cases} a_H d_1 + \frac{1-a_H}{2} + \frac{1}{2} + N(b) & \text{w.p.} \quad \pi \\ a_L d_1 + \frac{1-a_L}{2} + \frac{1}{2} + N(b) & \text{w.p.} \quad 1-\pi \end{cases},$$

- $\epsilon_H = \frac{a_H}{b}$, $\epsilon_L = \frac{a_L}{b}$
- Expected accuracy

$$\begin{aligned} e(a_L, a_H, b) &= \pi \cdot (2b^2 + ((2-a_H)/2)^2) \\ &+ (1-\pi) \cdot (2b^2 + ((2-a_L)/2)^2) \ . \end{aligned}$$

Option 2: offering only one seller the menu of contracts

Design the menu of contracts:

$$
\begin{aligned}
\min_{\{a_i, p_i, b_i,\ i \in \{H,L\}\}} \quad & \mathbb{E}(p) = \pi \cdot p_H + (1-\pi) \cdot p_L \\
\text{s.t.} \quad (IR) \quad & p_i \geq c(v_i, a_i/b),\ \ i \in \{H, L\} \\
(IC) \quad & p_i - c(v_i, a_i/b) \geq p_j - c(v_i, a_j/b),\ \ i, j \in \{H, L\} \\
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& 0 \leq a_i \leq 1, p_i \geq 0, b > 0,\ \ i \in \{H, L\}
\end{aligned}
$$

Simplifying the optimization

- (IR) Constraint is binding for users with high valuation.
- (IR) Constraint is redundant for users with low valuation.
- (IC) Constraint is binding for users with low valuation.

Broker's decision

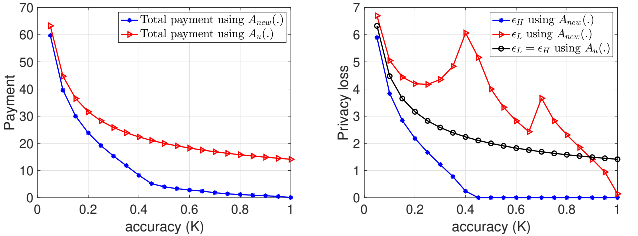
- $\mathcal{A}_u(D)$: a single contract $(a_H = a_L = 1)$ and $\begin{cases} b^* = \sqrt{K/2} \\ \epsilon^* = 1/b^* \\ p^* = c(v_H, 1/b^*) \end{cases}$
- $\mathcal{A}_{new}(D)$: a menu of contracts via **Option 1** or **Option 2**.

Broker's decision

- $\mathcal{A}_u(D)$: a single contract $(a_H = a_L = 1)$ and $\begin{cases} b^* = \sqrt{K/2} \\ \epsilon^* = 1/b^* \\ p^* = c(v_H, 1/b^*) \end{cases}$
- $\mathcal{A}_{new}(D)$: a menu of contracts via **Option 1** or **Option 2**.

A numerical example:

- $c(v_i, \epsilon_i) = v_i \cdot \epsilon_i,\ v_H = 5,\ v_L = 1,\ \pi = 0.5$



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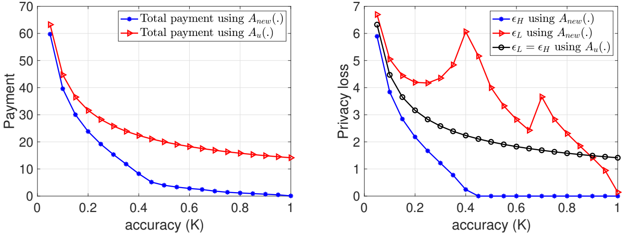

  - $\mathcal{A}_{new}(\cdot)$ lowers the payment significantly.
  - $\mathcal{A}_{new}(\cdot)$ can differentiate sellers of different types: $\epsilon_H < \epsilon_L$.

$$\begin{cases} K \leq 0.4 : \text{offer both sellers the menu of contracts} \\ 0.4 < K \leq 0.65 : \text{offer both sellers a single contract of low privacy type} \\ 0.65 < K : \text{offer a randomly seleted seller a single contract of low privacy type} \end{cases}$$

Conclusions

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- Generalization to non-linear queries and multi-dimensional data is available online.

Questions?

An example of nonlinear queries: polynomial queries

$$
\begin{aligned}
D &= (d_1, d_2)\\
Q(D) &= d_1^2 + d_1 \cdot d_2 + d_2^2\\
A_{new}(D) &= a_1 d_1^2 + a_{12} \cdot d_1 \cdot d_2 + a_2 d_2\\
&+ \frac{1-a_1}{2} + \frac{1-a_{12}}{2} + \frac{1-a_3}{2} + N(b)\\
\epsilon_1 &= \frac{a_1 + a_{12}}{b},\ \epsilon_2 = \frac{a_{12} + a_2}{b}\\
K &= (\frac{1-a_1}{2} + \frac{1-a_{12}}{2} + \frac{1-a_3}{2})^2 + 2b^2
\end{aligned}
$$